Course number and name: Partial Differential Equations
Credits: 6 ECTS (3 US credits)
Credit categorization: Upper-Level Undergraduate Mathematics / Applied Mathematics
Instructor: Giovanni Dalmasso
Office:
Email: giovanni.dalmasso@iqs.url.edu
Office hours:

Textbook:

Required:

Salsa, Sandro. *Partial Differential Equations in Action: From Modelling to Theory*. Springer, 2010. ISBN: 9788847007512. [*Preview with* Google Books]

Recommended supporting material:

- Supplementary Books:
    - Lawrence C. Evans. *Partial Differential Equations (2nd Edition)*. American Mathematical Society (AMS), 2010. ISBN: 978-0821849743.
    - Matthew P. Coleman and Vladislav Bukshtynov. *An Introduction to Partial Differential Equations with MATLAB (3rd Edition)*. CRC Press, Chapman & Hall, 2024. ISBN: 978-1032639383.
- Python Book:
    - Hans Petter Langtangen and Svein Linge. *Finite Difference Computing with PDEs: A Modern Software Approach*. Springer, 2017. ISBN: 978-3-319-55455-6.
- Online Resources:
    - SciPy: Python library for solving PDEs numerically. Accessible at: https://scipy.org/.
    - FEniCS: Open-source finite element library for solving boundary value problems. Accessible at: https://fenicsproject.org/.
    - Vedo: Python library for 3D visualization and interactivity. Accessible at: https://vedo.embl.es/.

Specific Course information:

Brief description:

This course offers an introduction to partial differential equations (PDEs), focusing on classical solution techniques and their applications in physics and engineering. Topics include first-order linear and nonlinear equations, the method of characteristics, second-order linear equations, and canonical forms of the heat, wave, and Laplace equations. Students will also learn about separation of variables, Fourier series, and integral transform methods. Analytical understanding will be complemented by hands-on Python-based exercises for visualizing and approximating solutions to PDEs.

Prerequisites or co-requisites:

A solid background in calculus (including multivariable calculus) and ordinary differential equations is required. Some familiarity with linear algebra and basic programming (in any language) is recommended.

Required material:

Lecture notes and problem sets will be provided. Students will use Python for selected assignments; installation instructions and support materials will be shared during the first week of class.

Course objectives and outcomes:

Course objectives:

1. Introduce the fundamental concepts and classifications of partial differential equations.
2. Develop analytical techniques for solving classical PDEs, including the heat, wave, and Laplace equations.
3. Build a conceptual understanding of boundary and initial value problems in various coordinate systems.
4. Explore methods such as separation of variables, Fourier series, and integral transforms.
5. Connect mathematical solutions to physical phenomena in engineering and the sciences.
6. Integrate computational tools based on Python for visualizing and approximating PDE solutions.

Course outcomes:

1. Classify and interpret different types of partial differential equations and their boundary conditions. *(ABET 1)*
2. Solve linear first- and second-order PDEs analytically using appropriate mathematical techniques. *(ABET 1)*
3. Construct Fourier series representations of functions and use them to solve PDEs. *(ABET 1, 7)*
4. Apply separation of variables to standard PDEs in rectangular and other simple domains. *(ABET 1, 2)*
5. Use characteristics to solve first-order nonlinear PDEs. *(ABET 1)*
6. Employ Python to simulate, visualize, and approximate solutions to classical PDEs. *(ABET 1, 6, 7)*
7. Communicate mathematical reasoning clearly in both written and computational formats. *(ABET 3)*
8. Work effectively in collaborative settings to solve PDE-related problems and share results. *(ABET 5)*
9. Reflect on the role of mathematical modeling in addressing real-world phenomena in science and engineering. *(ABET 2, 4)*

List of topics to be covered:

1. **Introduction to Partial Differential Equations**
   - Physical motivation and classification (elliptic, parabolic, hyperbolic)
   - Linear vs. nonlinear PDEs and well-posedness
2. **The Heat Equation**
   - Derivation and physical interpretation
   - Boundary and initial conditions
   - Uniqueness and the maximum principle
   - Fundamental solution and convolution
   - Inhomogeneous problems
3. **Laplace's and Poisson's Equations**
   - Derivation and applications
   - Fundamental solutions and Green's functions
   - Poisson's formula, Harnack's inequality, and Liouville's theorem
4. **The Wave Equation**

    - 1D and higher-dimensional cases
    - D’Alembert solution and spherical means
    - Kirchhoff’s formula and Minkowskian geometry
    - Geometric energy estimates
5. **Fourier Series and Separation of Variables**
    - Fourier series: convergence and orthogonality
    - Eigenvalue problems and Sturm–Liouville theory
    - Separation of variables for heat, wave, and Laplace equations
6. **Transform Methods**
    - Laplace transform and applications to initial-value problems
    - Fourier transform and its use in PDEs on unbounded domains
    - Plancherel’s theorem and inversion
7. **First-Order PDEs and Nonlinear Equations**
    - Method of characteristics
    - Burger’s equation and shock formation
8. **Introduction to Schrödinger’s Equation** *(optional/advanced)*
    - Fundamental solution and interpretation
    - Spectrum of the Laplace operator
9. **Lagrangian Field Theory** *(optional/advanced or project-based)*
    - Variational principles and physical motivation
    - Euler–Lagrange equations
10. **Python Applications in PDEs**
    - Visualization of solutions using Python (Matplotlib, SymPy, Vedo)
    - Discretization and finite difference schemes (SciPy, NumPy)
    - Numerical approximation of the heat and wave equations using finite difference methods (NumPy, SciPy) and finite element methods (FEniCS) - *(optional/advanced or project-based)*

Time distribution:

Week	Contact hours	Topic	Key dates
1	3	Introduction to PDEs Introduction to the heat equation	*No problem set (Pset)*
2	3	The heat equation: Uniqueness The heat equation: Weak maximum principle and introduction to the fundamental solution	Pset 1 due *No Pset*
3	3	The heat equation: Fundamental solution and the global Cauchy problema (Simple heat equation solution visualization with matplotlib or Vedo) Laplace's and Poisson's equations	Pset 2 due *No Pset*
4	3	Poisson's equation: Fundamental solution Poisson's equation: Green functions	Pset 3 due *No Pset*
5	3	Poisson's equation: Poisson's formula, Harnack's inequality, and Liouville's theorem Introduction to the wave equation	Pset 4 due Pset 5 due
6	3	The wave equation: The method of spherical means The wave equation: Kirchhoff's formula and Minkowskian geometry (Simulating wave propagation with Vedo animations)	Pset 6 due *No Pset*
7	3	The wave equation: Geometric energy estimates **Midterm Exam 1**	*No Pset*
8	3	The wave equation: Geometric energy estimates (cont.) Classification of second order equations	*No Pset* Pset 7 due
9	3	Introduction to the Fourier transform Introduction to the Fourier transform (cont.)	*No Pset* Pset 8 due
10	3	Fourier inversion and Plancherel's theorem Introduction to Schrödinger's equation (Use FEniCS to solve Laplace or Schrödinger numerically on a 2D domain)	*No Pset*
11	3	Introduction to Schrödinger's equation (cont.) Introduction to Lagrangian field theories / student-led seminar	Pset 9 due *Optional* Prob due
12	3	Introduction to Lagrangian field theories (cont.) Introduction to Lagrangian field theories (cont.)	Pset 10 due *No Pset*
13	3	Transport equations and Burger's equations Case Studies in PDEs: Connecting Equations to Real Systems	Pset 11 due *No Pset*

14	3	**Final Exam**	*No Pset*

Assessment structure:

Methods of Evaluation	Weight	Date/freq.	Description
Midterm exam	30%	One per course	1 midterm exam lasting 1,5 hours
Final exam	Up to 40%	End of semester	Cumulative exam allowing to recover contents of the midterm exam
Following up activities	30%	As described in the table distribution	Homework, Quizzes, Projects

PERFORMANCE INDICATORS AND GRADING

One written midterm exam will be administered around the middle of the term, as indicated in the course schedule. A comprehensive final exam will be held at the end of the semester.

The final exam may be used to **recover a failed midterm** (i.e., to replace a failing grade), but **not to improve an already passing score**. Its purpose is to ensure that students have achieved the essential learning outcomes, particularly if earlier performance was insufficient.

Additional components contributing to the final grade may include problem sets, optional assignments, and participation, as specified in the detailed grading breakdown.

Homework, Quizzes, Project	30%
Midterm Exam	30%
Final Exam	40%

HOMEWORK POLICY

Homework assignments are due one week after they are assigned, unless otherwise specified. Each assignment is scheduled to allow an intervening class session where students may ask questions and clarify doubts.

Late submissions will **not be accepted** and will receive a grade of **zero**, except in cases of documented emergencies or approved absences. Students are encouraged to start early and seek help during office hours or in class if difficulties arise.

CLASS STRUCTURE

Lectures will serve as the primary mode of instruction. Students are expected to attend all sessions and actively engage in class discussions. Homework assignments will be reviewed and discussed during lectures, and students may be asked to solve problems individually or collaboratively during class time.

To get the most out of each session, students are strongly encouraged to **read the relevant material in advance**. Active participation and preparation are key to mastering the concepts covered in this course.

ACADEMIC INTEGRITY

All work submitted in this course must adhere strictly to the principles of academic honesty and integrity. **Cheating, plagiarism, unauthorized collaboration, or any form of academic dishonesty** will not be tolerated and will be addressed in accordance with the **disciplinary regulations of IQS**.

Students are responsible for understanding what constitutes academic misconduct. When in doubt, it is always best to ask the instructor for clarification before submitting work.

DOCUMENT HISTORY

PREVIOUS REVISIONS

Month, Year. Prof. name.

LAST REVISION

Month, Year. Prof. name.